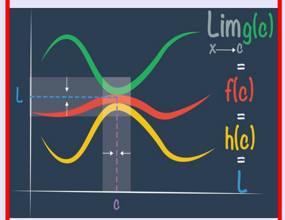
Math 261

Fall 2022

Lecture 26



Math 261
Class Quiz 13

Name:____________________

No Work ⇔ No Points

Use Pencil Only ⇔ Be Neat & Organized

1. (6 points) At what point(s) is the tangent line to the curve $y^3 = 2x^2$ perpendicular to the line $x + 2y - 2 = 0$.

Tan. line $\perp$

$2y = -x + 2 \to y = \frac{-1}{2}x + 1$

$m = \frac{-1}{2}$

$m_{\text{Tan. line}} = 2$

$3y^2 \frac{dy}{dx} = 4x$

$\frac{dy}{dx} = \frac{4x}{3y^2}$

$\frac{4x}{3y^2} = 2$

$2x = 3y^2$

$x = \frac{3y^2}{2}$

$y^3 = 2\left(\frac{3y^2}{2}\right)^2$

$y^3 = \frac{9y^4}{2}$

$9y^4 - 2y^3 = 0$

$y^3(9y - 2) = 0$

1. $(0,0), (?, \frac{2}{9})$

2. (8 points) Find all the points on the graph of $x^2 + xy + y^2 = 3$ have horizontal tangent line.

$m = 0$

$2x + 1 \cdot y + x \cdot \frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$

$-2x - y = 0 \quad \boxed{y = -2x}$

$x^2 + x(-2x) + (-2x)^2 = 3$

$x^2 - 2x^2 + 4x^2 = 3$

$3x^2 = 3$

$x^2 = 1$

$(1, ?)(-1, ?)$

2. ________

3. (6 points) If $f(x)$ is an odd function, show that $f'(x)$ is an even function.

$f(-x) = -f(x)$

$\frac{d}{dx}[f(-x)] = \frac{d}{dx}[-f(x)]$

$f'(-x) \cdot \cancel{-}1 = \cancel{-}f'(x)$

$f'(-x) = f'(x)$

$f'(x)$ is an even function.

3. ________

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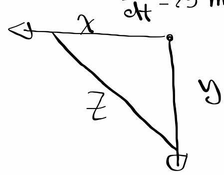
Two cars start moving from the same point.
one going South at 60 mi/hr, and the other one going west at 25 mph.
At what rate is the distance between them changing 2 hrs later?

$\frac{dx}{dt} = 25$ mph



$\frac{dy}{dt} = 60$ mph

$$z^2 = x^2 + y^2$$
$$\vdots$$
$$z\frac{dz}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$$
$$130\frac{dz}{dt} = 50\cdot 25 + 120\cdot 60$$

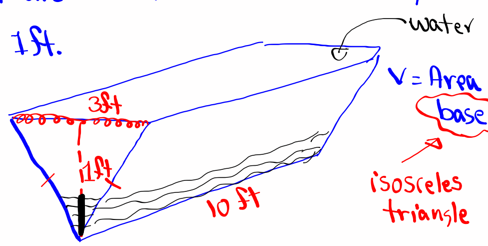
In 2 hrs $y = 120$, $x = 50$

$$z^2 = x^2 + y^2$$
$$z^2 = 50^2 + 120^2$$
$$z = \sqrt{50^2 + 120^2} = 130$$

$$\frac{dz}{dt} = \frac{50\cdot 25 + 120\cdot 60}{130}$$

$$\boxed{\frac{dz}{dt} = ?}$$

A trough is 10 ft long, and its ends have the shape of isosceles triangles that are 3 ft across at the top and height of 1 ft.



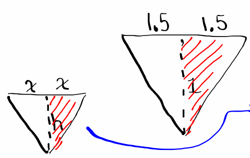
V = Area of the base · height

isosceles triangle → base; height = 10

If it is being filled with water at the rate of 12 ft³/min. $\frac{dV}{dt} = 12$ ft³/min.

How fast is the water level rising when the water is 6 inches deep?

$h = 6$ in. $= \frac{1}{2}$ ft



$\frac{x}{h} = \frac{1.5}{1}$ $\quad x = 1.5h$ $\quad \boxed{x = \frac{3}{2}h}$

Area $= \frac{\text{base}\cdot\text{height}}{2} = \frac{2x\cdot h}{2}$

Area $= xh$ $\rightarrow$ $\boxed{\text{Area} = \frac{3}{2}h^2}$

$$V = \frac{3}{2}h^2 \cdot 10 \qquad V = 15h^2$$

$$\frac{dV}{dt} = 15 \cdot 2h \cdot \frac{dh}{dt}$$

$$12 = 30 \cdot \frac{1}{2} \cdot \frac{dh}{dt}$$

$$12 = 15\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{5} \text{ ft/min.}$$

Two sides of a triangle are 12m & 15m.

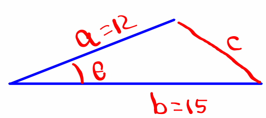
The angle between them is increasing at

2°/min.

How fast is the third side increasing when

that angle is 60°?

$$\frac{d\theta}{dt} = 2°/\text{min.}$$



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$c^2 = 12^2 + 15^2 - 2(12)(15)\cos\theta$$

$$2c\frac{dc}{dt} = 0 - 360 \cdot -\sin\theta \cdot \frac{d\theta}{dt}$$

When $\theta = 60°$

$$c^2 = 12^2 + 15^2 - 2(12)(15)\cdot\cos 60°$$

$$= 144 + 225 - 360 \cdot \frac{1}{2}$$

$$c^2 = 189 \qquad \boxed{c \approx 13.75}$$

$$2\sqrt{189}\frac{dc}{dt} = 360 \cdot \sin 60° \cdot 2°$$

$$\cancel{2}\sqrt{189}\frac{dc}{dt} = \cancel{360}^{2} \cdot \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\pi}{\cancel{90}}$$

$180° = \pi$ rad.

$1° = \frac{\pi}{180}$ Rad

$2° = \pi/90$ Rad.

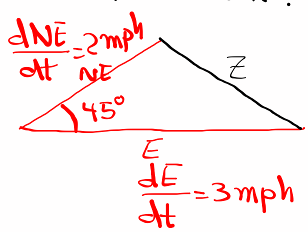
$$\frac{dc}{dt} = \frac{\pi\sqrt{3}}{\sqrt{189}} = \frac{\pi}{\sqrt{63}}$$

m/min.

Two people start from the same point

One goes east @ 3 mi/hr, the other one goes northeast @ 2 mi/hr.

How fast is the distance between them changing after 15 minutes?



$Z^2 = (NE)^2 + E^2 - 2(NE)(E)\cdot\cos 45°$

$Z^2 = (NE)^2 + E^2 - \cancel{2}(NE)(E)\cdot\frac{\sqrt{2}}{\cancel{2}}$

Find Z.

$2Z\frac{dZ}{dt} = 2(NE)\cdot\frac{dNE}{dt} + 2E\frac{dE}{dt} - \sqrt{2}\left(\frac{dNE}{dt}\cdot E + NE\cdot\frac{dE}{dt}\right)$

$\frac{dZ}{dt} = ?$

In 15 minutes,

$E = \frac{3}{4}$

$NE = \frac{2}{4} = \frac{1}{2}$

Estimate $(1.01)^6$

$f(x) = x^6$

$a = 1$

$f(1) = 1^6 = 1$

$f'(x) = 6x^5$

$f'(1) = 6$

$L(x) \approx f(a) + f'(a)(x-a)$

$f(x) \approx 1 + 6(x-1)$

Near 1.

$(1.01)^6 \approx 1 + 6(1.01 - 1)$

$\approx 1 + 6(.01)$

$(1.01)^6 \approx \boxed{1.06}$

from calc. $\boxed{1.06}152\ldots$

Use linear approximation to show

$\sec .08 \approx 1$

From Calc.

$\sec .08 = \dfrac{1}{\cos .08}$

$\approx 1.0032\ldots$

$f(x) = \sec x$

$a = 0$

$L(x) = f(a) + f'(a)(x-a)$

$f(0) = \sec 0$

$= 1$

$= 1 + 0(x-0)$

$L(x) = 1$

$f'(x) = \sec x \tan x$

$\sec .08 \approx 1$

$f'(0) = \sec 0 \cdot \tan 0 = 0$